SKIN FRICTION AND HEAT TRANSFER IN TURBULENT FLOW OF STRUCTURALLY VISCOUS FLUIDS

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Relations which contain no new empirical turbulence constants are proposed for calculation of the resistance coefficient and the Nusselt number in turbulent flow of structurally viscous fluids.

The hydrodynamics and heat transfer of turbulent flows of non-Newtonian fluids in circular pipes were discussed in [1, 2]. There the authors, using a rheological power equation and making a number of assumptions, proposed formulas for the velocity distributions and the resistance coefficients which contained new empirical constants determined from experiments on the flow of non-Newtonian liquids and depending on the "degree of non-Newtonian-ness" (the exponent in the rheological power equation).

For the structurally viscous liquids examined in these references, however, where the rheological properties of the liquids depend only on the thermodynamic state parameters and the tangential shear stress, it is not necessary to introduce any new empirical turbulence constants. It is sufficient to proceed from the following two well-known facts:

1) The thickness of the viscous sublayer is small compared with the pipe diameter, and the shear stress in it, and therefore the fluidity, are practically equal to their values at the wall, τ_w and φ_w.

2) In the turbulent core of the stream, the Reynolds stresses do not depend on the molecular viscosity, i.e., (when $\mu > 0$) the turbulent shear stresses are self-similar relative to the dependence $\varphi(\tau)$.

Further, since in the region $y < y_1$, $\tau \approx \tau_w$ and $\varphi \approx$ const, the criterion describing the stability of the viscous sublayer of Newtonian liquids [3] also holds for structurally viscous liquids, i.e.,

$$(v^* y/\nu)_1 = v^* y_1 \rho \varphi_w = \text{const},$$

where y_1 is the thickness of the viscous sublayer, ρ is the density of the liquid, and φ_w is the fluidity for shear stress τ_w at the wall.

Velocity distribution and resistance coefficient. From the above it follows that the velocity distribution in a turbulent stream of a structurally viscous liquid, expressed in dimensionless coordinates, will coincide with the universal velocity profile of Newtonian liquids, if the dimensionless coordinate is defined with respect to the fluidity at the wall:

$$w/v^* = \eta_w \quad \text{for } 0 \leqslant \eta_w \leqslant 6,$$

$$w/v^* = -3.05 + 5 \ln \eta_w \quad \text{for } 6 \leqslant \eta_w \leqslant 30,$$
$$w/v^* = 5.5 + 2.5 \ln \eta_w \quad \text{for } \eta_w > 30, \tag{1}$$

where

$$\eta_w = v^* y \rho \varphi_w. \tag{2}$$

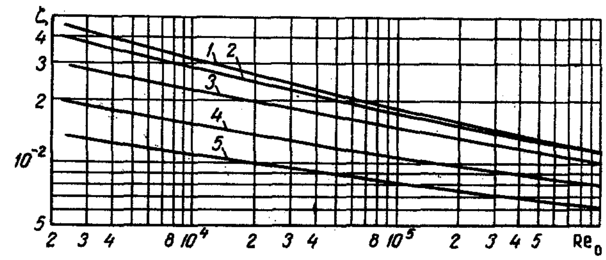

Fig. 1. Dependence of resistance coefficient ζ on Re_0 with $\beta = 0$ (1), 10^2 (2), 10^3 (3), 10^4 (4), and 10^5 (5).

Similarly, the resistance coefficient in a turbulent flow, ζ, may be determined from the formulas for Newtonian liquids, where the wall value of the viscosity of the liquid enters into the Re number.

It is inconvenient, however, to use the relation for the resistance coefficient in the form $\zeta(Re_w)$, since for given values of the mean velocity $\bar{w}$ and pipe diameter D, we require an interation method to determine the quantity ζ. (By assigning first the value of τ_w, we determine φ_w and Re_w, and then, having determined ζ, we refine the value of $\tau_w = (\zeta/8)\rho \bar{w}^2$, etc.) For structurally viscous liquids with a linear law of fluidity [4]

$$\varphi = \varphi_0\left(1 + \frac{\Theta}{\varphi_0}\tau_w\right) = \varphi_0\left(1 + \frac{\zeta\Theta}{8\varphi_0}\rho \bar{w}^2\right), \tag{3}$$

where φ_0 is the fluidity at $\tau_w = 0$; Θ is the coefficient of structural stability of the liquid.

In this case it is convenient to introduce the following dimensionless parameters:

$$Re_0 = v^* D \rho \varphi_0, \tag{4}$$

$$\beta = \frac{\Theta}{\varphi_0}\rho \bar{w}^2. \tag{5}$$

Substituting the quantity

$$Re_w = Re_0\left(1 + \frac{\zeta}{8}\beta\right) \tag{6}$$

in the formula for determining the resistance coefficient of Newtonian liquids [5], we obtain

$$\frac{1}{\sqrt{\zeta}} = 0.88 \ln\left[Re_0\sqrt{\zeta}\left(1 + \frac{\zeta}{8}\beta\right)\right] - 0.9. \tag{7}$$

The dependence $\zeta(\mathrm{Re}_0, \beta)$ for turbulent flow is shown in Fig. 1. With $\beta > 100$ formula (7) may be approximated by the expression

$$\zeta = \zeta_0 - 0.11\,\mathrm{Re}_0^{-0.3}(\ln\beta - 2), \tag{8}$$

where ζ_0 is the resistance coefficient as determined by the formulas for Newtonian liquids:

$$\zeta_0 = 0.316\,\mathrm{Re}_0^{-0.25} \quad \text{for} \quad 10^4 \leqslant \mathrm{Re}_0 \leqslant 10^5$$

and

$$\zeta_0 = 0.0032 + 0.221/\mathrm{Re}^{0.237} \quad \text{for} \quad \mathrm{Re}_0 > 10^5.$$

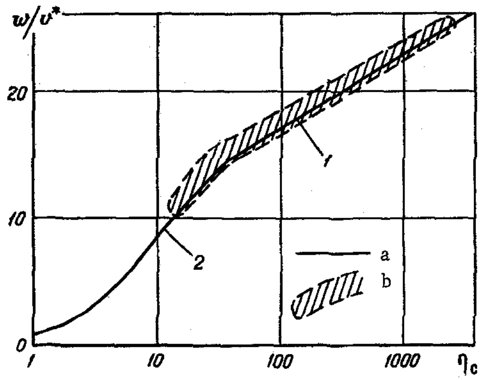

Fig. 2. Velocity distribution: a) universal velocity profile; b) experimental data of Clapp [2]; 1) $w/v^* = 5.5 + 2.5\ln\eta_w$; 2) $w/v^* = -3.05 + 5\ln\eta_w$.

It should be borne in mind that the number $\mathrm{Re}_{0.cr}$, corresponding to transition from the laminar to the turbulent regime, will depend on the quantity β.

Heat transfer. In calculating the heat transfer coefficient for liquids with structural viscosity, we may use the usual formulas for Newtonian liquids, inserting in them the values of the Reynolds and Prandtl numbers computed from the viscosity at the wall. However, in the majority of cases, the Prandtl numbers are large ($\mathrm{Pr} > 100$) for structurally viscous liquids, and the entire thermal resistance is practically concentrated inside the viscous sublayer. In this case, to determine the dimensionless heat transfer coefficient, under the assumption that the liquid properties are independent of temperature, we may use the approximate formula of [5]:

$$\mathrm{Nu} \approx 2\left[\int_{\xi_1}^{1} \frac{d\zeta}{1 + \mathrm{Pr}_w \mu_T/\mu_w}\right]^{-1}, \tag{9}$$

where $\xi_1 = 1 - y/R$ is the dimensionless boundary of the viscous sublayer; μ_T is the turbulent viscosity; Pr_w and μ_w are the Prandtl number and the viscosity of the liquid at the pipe wall. When $\mathrm{Pr} \to \infty$ this formula may be transformed to the form

$$\mathrm{Nu} = 0.035\,\mathrm{Re}_{\bar{w}}\sqrt{\zeta}\,\mathrm{Pr}_w^{1/4} = 0.035\,\mathrm{Re}_0\mathrm{Pr}_0^{1/4}\sqrt{\zeta}\left(1 + \frac{\zeta}{8}\beta\right)^{3/4}, \tag{10}$$

where Re_0 and Pr_0 are determined from the zero fluidity φ_0, and the resistance coefficient ζ—from (7).

Comparison with experimental data. It should be noted that the majority of authors, using the rheological power equation

$$\tau = k\left(\frac{dw}{dy}\right)^n, \tag{11}$$

reduce the experimental data in the form $\zeta = f_1(\mathrm{Re}')$ and $w/v^* = f_2(y+)$, where

$$\mathrm{Re}' = \bar{w}^{2-n}\rho D^n \Big/ k\left(\frac{3n+1}{4n}\right)^n 8^{n-1}, \tag{12}$$

$$y^+ = v^{*2-n}\rho y^n/k. \tag{13}$$

Comparison of (7) and (10) with the data available in the literature is therefore difficult.

However, we can show that η_w and y^+ are connected by the relation

$$\eta_w = (y^+)^{1/n}(\varphi_w/\varphi_w^a)^{1/n}, \tag{14}$$

where φ_w^a is the apparent fluidity ($\varphi_w^a = 8\bar{w}/D\tau_w$ for laminar flow in a circular pipe). For structurally viscous liquids with a linear fluidity law, the relation between the true and the apparent fluidity is determined by the simple expression [4]

$$\varphi = \varphi_a\left(1 + \frac{\theta}{\varphi_0}\tau_w\right)\Big/\left(1 + 1.25\frac{\theta}{\varphi_0}\tau_w\right). \tag{15}$$

The experimental data of Clapp [2] on velocity distribution, converted using (14) and (15), are shown in Fig. 2. The agreement between these data and the universal profile may be considered good. The agreement in practice of velocity profiles in the turbulent core of a stream for Newtonian and structurally viscous liquids in the coordinates $w/\bar{w} = f(y|R)$ [7] also confirms the correctness of relations (1) and (2).

There are few experimental data on heat transfer in turbulent flows of non-Newtonian liquids. We know of only Clapp's tests [2] of heat transfer with carbopol solutions of two different concentrations in a comparatively small range of variation of the parameters ($5 \cdot 10^3 \leq \mathrm{Re}_w \leq 2.7 \cdot 10^4$ and $80 \leq \mathrm{Pr}_w \leq 104$). These data are in satisfactory agreement with calculations acording to (10).

It should be noted that in a number of tests we observed anomalously low values of the resistance coefficient in a turbulent flow of water with small additions of soluble high-molecular compounds [1, 7, 8]. The causes of this phenomenon are not known at present. The hypothesis has been expressed that it may be connected with the formation in these solutions of a supra molecular structure [9] or with the influence of elastic properties of the liquid [7]. Of course, these solutions cannot be related to the type of liquids with structural viscosity.

NOTATION

τ—shear stresses; φ—fluidity of liquid; y—distance from surface; v^*—dynamic viscosity; ν—kinematic viscosity; ρ—density; w—flow velocity of liquid; $\bar{w}$—mean mass flow velocity; η—dimensionless coordinate; ζ—resistance coefficient; R—pipe radius; D—pipe diameter;

Θ—coefficient of structural stability of liquid; β—dimensionless parameter; $\xi = 1 - y/R$—dimensionless coordinate; μ_T—turbulent viscosity. Subscripts: 0—at $\tau_w = 0$; 1—at edge of viscous sublayer; w—at tube wall.

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